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# The single-blow transient testing technique for plate–fin heat exchangers

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**Abstract**

A new model of the single-blow problem is proposed, considering the lateral heat conduction resistance along the fins, the axial heat conduction along the separating plates and the axial thermal dispersion in the fluid. For plate–fin heat exchangers made up of stainless steel, the effect of the lateral heat conduction resistance along the fins can usually not be neglected. This effect is taken into account by solving the temperature dynamics in the fluid, separating plates and fins simultaneously. The axial dispersion model is used to take flow maldistribution in plate–fin heat exchangers into account. The effect of the axial heat conduction in separating plates is also considered. The governing equation system is solved by means of Laplace transform and numerical inverse transform algorithms. The investigation confirms: for plate–fin heat exchangers of aluminium the effect of the lateral heat conduction resistance of fins can usually be neglected because of their high fin efficiency. However, the assumption of uniform porous medium is not valid if the plate–fin heat exchangers are made up of stainless steel. In such a case the heat conduction resistance of fins has significant influence on the outlet fluid temperature variation. 

*Keywords:* Heat exchangers; Measurement techniques; Transient

## 1. Introduction

Single-blow transient testing technique has been widely used for the determination of heat transfer coefficients in compact heat exchangers [1]. The name "single-blow" means that the experiment uses only one fluid stream and heat transfer occurs only between the fluid and the heat transfer surface along which the fluid passes. The heat transfer surface of the test core to be measured is usually considered as a uniformly distributed porous matrix. With single-blow technique one can directly obtain the convective heat transfer coefficient between the heat transfer surface and the fluid.

In the single-blow experiment, a fluid flows steadily through a test core. Initially the fluid and the test core have the same uniform temperature. Then a fluid temperature variation is introduced at the inlet and the local fluid temperature histories (usually at inlet and outlet of the test core) are measured continuously. These data are then compared with the theoretical result to obtain the corresponding heat transfer coefficient between the heat transfer surface and the fluid. Obviously, one of the prerequisites for the single-blow technique is that the mathematical model should coincide with the real heat transfer process in the experiment as closely as possible.

In the original mathematical model of single-blow problem a step change in inlet fluid temperature and a uniform porous medium without longitudinal heat conduction and dispersion are considered. Its analytical solution were provided in different forms by Anzelius [2], Nusselt [3], Hausen [4] and Schumann [5]. Schumann's solution was first used as the basis for a transient technique by Furnas [6]. The effect of axial heat conduction in porous materials was investigated numerically by Creswick [7] and Howard [8]. Recognising that the actual

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**Nomenclature**

$A_c$ test core free-flow area, m$^2$
$A_f$ total cross-sectional area of the fins perpendicular to the fin height coordinate, m$^2$
$A_p$ total cross-sectional area of the separating plates perpendicular to the flow direction, m$^2$
$B$ ratio of heat capacity of fluid to total wall heat capacity in the test core, $B = C/(C_p + C_f)$, dimensionless
$Bi$ Biot number, $Bi = h\alpha_f F_f/(A_f k_f)$, dimensionless
$C$ heat capacity of fluid in the test core, J/K
$\dot{C}$ thermal flow rate, W/K
$C_f$ heat capacity of fins in the test core, J/K
$C_p$ heat capacity of separating plates in the test core, J/K
$D$ axial dispersion coefficient, W/m K
$F$ heat transfer area, m$^2$
$h$ fin height, m
$k$ heat conductivity, W/m K
$K_f$ dimensionless lateral heat conductivity of fins, $K_f = A_f k_f/(h\dot{C})$, dimensionless
$K_p$ dimensionless lateral heat conductivity of separating plates, $K_p = A_p k_p/(L\dot{C})$, dimensionless
$L$ length of test core, m
NTU number of transfer units, $\mathrm{NTU} = [(\alpha F)_p + (\alpha F)_f]/\dot{C}$, dimensionless
$Pe$ axial dispersive Peclet number, $Pe = \dot{C}L/(A_c D)$, dimensionless
$s$ complex parameter in the Laplace transform, dimensionless; also fin space, m
$T$ temperature, K
$\bar{x}$ longitudinal coordinate in flow direction, $\bar{x} = x/L$, dimensionless
$\bar{y}$ lateral coordinate along the fin surface and perpendicular to the flow direction, $\bar{y} = y/h$, dimensionless
$\alpha$ heat transfer coefficient, W/m$^2$ K
$\delta_f$ fin thickness, m
$\delta_p$ thickness of separating plate, m
$\zeta$ parameter, $\zeta = C_f/(C_p + C_f)$, dimensionless
$\eta$ parameter defined in Eq. (11)
$\theta$ dimensionless fluid temperature, $\theta = (T - T_0)/(T_{max} - T_0)$, dimensionless
$\theta_f, \theta_p$ dimensionless temperatures of fins and separating plates, $\theta_f = (T_f - T_0)/(T_{max} - T_0)$, $\theta_p = (T_p - T_0)/(T_{max} - T_0)$
$\xi$ parameter, $\xi = (\alpha F)_f/[(\alpha F)_p + (\alpha F)_f]$, dimensionless
$\bar{\tau}$ dimensionless time, $\bar{\tau} = \tau\dot{C}/(C_p + C_f)$, dimensionless

*Subscripts*
0 initial value
f fin
in inlet
p separating plate

profile of the inlet fluid temperature can usually be regarded as an exponential function, Liang and Yang [9] obtained the analytical temperature response solution for the exponential inlet fluid temperature change. Based on the numerical results using finite differential method, Cai et al. [10] extended Liang and Yang's analysis with an empirical formula obtained from numerical results to include the effect of the axial heat conduction in the solid material of the test core. They concluded that the effect of axial heat conduction should be considered for the case of $\mathrm{NTU} \geqslant 3$ and $K_p\ \mathrm{NTU} \geqslant 0.06$. More details about the single-blow technique can be found in the literature [11–14]. The new development in mathematical models of single-blow problems was made by Roetzel and Luo [15], in which the flow maldistribution is taken into account with an axial dispersion term in the energy equation of the fluid. Subsequently, the pulse testing technique applying an arbitrary pulse temperature variation as inlet condition was developed by Zhou et al. [16] to determine the heat transfer coefficient and the axial dispersion coefficient simultaneously. More recently, a conduction/dispersion model was applied to the single-blow technique with arbitrary pulse inlet condition by Luo et al. [17], which takes both the axial heat conduction in the wall material and the axial heat dispersion in the fluid into account.

Some research work dealt with the effect of non-uniform lateral temperature distribution on the outlet temperature response. Using perturbation method, Zhou and Cai solved the single-blow problem considering the lateral heat conduction resistance of the wall [18]. The non-uniform lateral temperature distribution in side bars of a plate–fin heat exchanger was investigated by Luo et al. [19]. They used an approximate method to calculate the amount of the thermal capacity of the side bars which should be added into the thermal capacity of the main porous solid. Their method was also applied to the measurement of heat transfer coefficients and axial dispersion coefficients in plate heat exchangers using temperature oscillation method [20]. Zhang et al. [21] used a similar method to take non-uniform thermal capacity distribution in heat exchangers into consideration.

Recently a new mathematical model which accounts for the effect of the lateral heat conduction resistance along the fins in fin height direction is proposed by Luo and Roetzel [22], in which the unsteady lateral heat conduction in fins is coupled with the energy equations of the fluid and the separating plates. This model is further developed in the present work to take the axial heat conduction in separating plates and the axial heat dispersion in the fluid into account.

## 2. Mathematical description

Consider a fluid flowing steadily through a plate–fin heat exchanger, whose temperature at the entrance to the exchanger is initially constant and then varies with time when the test begins. It is assumed that the heat transfer coefficient and specific heat capacities of the fluid and the wall material are constant. There is no heat conduction resistance perpendicular to the plate and fin surfaces and no heat loss to the environment. The heat conduction in fins in the flow direction can be neglected. To simplify the problem we will use the axial dispersion model to account for the flow non-uniformity. The configuration of the plate–fin heat exchanger surface is shown in Fig. 1. The mathematical description can be obtained from energy balances of the fluid, separating plates and fins. The resulting system of dimensionless partial differential equations is summarized below together with the initial and boundary conditions:

$$B\frac{\partial\theta}{\partial\bar{\tau}}+\frac{\partial\theta}{\partial\bar{x}}-\frac{1}{Pe}\frac{\partial^2\theta}{\partial\bar{x}^2}=\mathrm{NTU}\left[(1-\xi)\theta_\mathrm{p}+\xi\int_0^1\theta_\mathrm{f}\,\mathrm{d}\bar{y}-\theta\right], \quad (1)$$

$$(1-\zeta)\frac{\partial\theta_\mathrm{p}}{\partial\bar{\tau}}-K_\mathrm{p}\frac{\partial^2\theta_\mathrm{p}}{\partial\bar{x}^2}=(1-\xi)\mathrm{NTU}(\theta-\theta_\mathrm{p})+2K_\mathrm{f}\left.\frac{\partial\theta_\mathrm{f}}{\partial\bar{y}}\right|_{\bar{y}=0}, \quad (2)$$

$$\zeta\frac{\partial\theta_\mathrm{f}}{\partial\bar{\tau}}=K_\mathrm{f}\frac{\partial^2\theta_\mathrm{f}}{\partial\bar{y}^2}+\xi\mathrm{NTU}(\theta-\theta_\mathrm{f}), \quad (3)$$

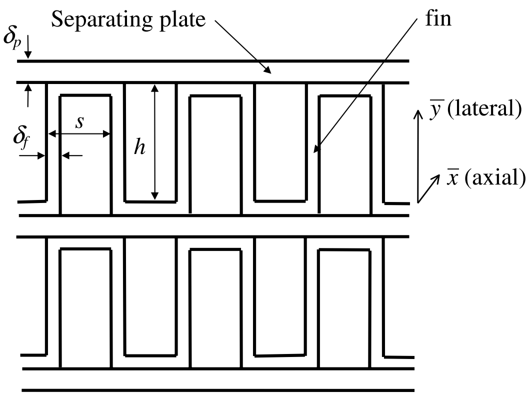


Fig. 1. Plate–fin heat exchanger.

$$\bar{\tau}=0: \quad \theta=\theta_\mathrm{p}=\theta_\mathrm{f}=0, \quad (4)$$

$$\bar{x}=0: \quad \theta-\frac{1}{Pe}\frac{\partial\theta}{\partial\bar{x}}=\theta_\mathrm{in}(\bar{\tau}), \quad \frac{\partial\theta_\mathrm{p}}{\partial\bar{x}}=0, \quad (5)$$

$$\bar{x}=1: \quad \frac{\partial\theta}{\partial\bar{x}}=\frac{\partial\theta_\mathrm{p}}{\partial\bar{x}}=0, \quad (6)$$

$$\bar{y}=0 \quad \text{and} \quad \bar{y}=1: \quad \theta_\mathrm{f}=\theta_\mathrm{p} \quad (7)$$

in which $\theta, \theta_\mathrm{f}$ and $\theta_\mathrm{p}$ are dimensionless temperatures of fluid, fins and separating plates, respectively. The axial dispersive Peclet number $Pe$ in Eq. (1) represents the flow maldistribution which must either be evaluated experimentally or deduced from a similar system. Eqs. (5) and (6) for the fluid are known as Danckwerts' boundary condition [23] which results from the conservation law of energy. It indicates an additional assumption that there is no axial dispersion before the inlet sections and after the outlet sections of the apparatus being investigated. The inlet fluid temperature $\theta_\mathrm{in}(\bar{\tau})$ can be an arbitrary function of time.

## 3. Analytical solution in Laplace plane

The above governing equation system is solved by means of Laplace transform. In the Laplace plane the fin temperature can be solved alone by taking $\theta$ and $\theta_\mathrm{p}$ as parameters, which yields

$$\tilde{\theta}_\mathrm{f}=[\cosh(r\bar{y})-\tanh(r/2)\sinh(r\bar{y})]\times\left(\tilde{\theta}_\mathrm{p}-\frac{\xi\mathrm{NTU}}{s\zeta+\xi\mathrm{NTU}}\tilde{\theta}\right)+\frac{\xi\mathrm{NTU}}{s\zeta+\xi\mathrm{NTU}}\tilde{\theta}, \quad (8)$$

$$\int_0^1\tilde{\theta}_\mathrm{f}\,\mathrm{d}\bar{y}=\eta\tilde{\theta}_\mathrm{p}+(1-\eta)\frac{\xi\mathrm{NTU}}{s\zeta+\xi\mathrm{NTU}}\tilde{\theta}, \quad (9)$$

$$2K_\mathrm{f}\left.\frac{\mathrm{d}\tilde{\theta}_\mathrm{f}}{\mathrm{d}\bar{y}}\right|_{\bar{y}=0}=-\eta\left[(s\zeta+\xi\mathrm{NTU})\tilde{\theta}_\mathrm{p}-\xi\mathrm{NTU}\tilde{\theta}\right], \quad (10)$$

where

$$\eta=\tanh(r/2)/(r/2), \quad (11)$$

$$r=\sqrt{(s\zeta+\xi\mathrm{NTU})/K_\mathrm{f}}. \quad (12)$$

Substituting Eqs. (9) and (10) into the Laplace transforms of Eqs. (1) and (2), one obtains

$$\frac{1}{Pe}\frac{\mathrm{d}^2\tilde{\theta}}{\mathrm{d}\bar{x}^2}=\frac{\mathrm{d}\tilde{\theta}}{\mathrm{d}\bar{x}}+A_1\tilde{\theta}-A_3\tilde{\theta}_\mathrm{p}, \quad (13)$$

$$K_\mathrm{p}\frac{\mathrm{d}^2\tilde{\theta}_\mathrm{p}}{\mathrm{d}\bar{x}^2}=A_2\tilde{\theta}_\mathrm{p}-A_3\tilde{\theta}, \quad (14)$$

$$\bar{x} = 0: \quad \tilde{\theta} - \frac{1}{Pe}\frac{\mathrm{d}\tilde{\theta}}{\mathrm{d}\bar{x}} = \tilde{\theta}_{\mathrm{in}}(s), \quad \frac{\mathrm{d}\tilde{\theta}_{\mathrm{p}}}{\mathrm{d}\bar{x}} = 0, \tag{15}$$

$$\bar{x} = 1: \quad \frac{\mathrm{d}\tilde{\theta}}{\mathrm{d}\bar{x}} = \frac{\mathrm{d}\tilde{\theta}_{\mathrm{p}}}{\mathrm{d}\bar{x}} = 0. \tag{16}$$

The complex coefficients $A_i$ $(i = 1, 2, 3)$ in Eqs. (13) and (14) are given as

$$A_1 = sB + (1-\xi)\mathrm{NTU} + \xi\mathrm{NTU}\frac{s\zeta + \eta\xi\mathrm{NTU}}{s\zeta + \xi\mathrm{NTU}}, \tag{17}$$

$$A_2 = s(1-\zeta) + (1-\xi)\mathrm{NTU} + \eta(s\zeta + \xi\mathrm{NTU}), \tag{18}$$

$$A_3 = (1-\xi)\mathrm{NTU} + \eta\xi\mathrm{NTU}. \tag{19}$$

The general form of the solution can be expressed in the matrix form

$$\widetilde{\mathbf{Y}} = \mathbf{U}e^{\mathbf{L}\bar{x}}\mathbf{D}, \tag{20}$$

where

$$\widetilde{\mathbf{Y}} = \left[\frac{\mathrm{d}\tilde{\theta}}{\mathrm{d}\bar{x}}, \frac{\mathrm{d}\tilde{\theta}_{\mathrm{p}}}{\mathrm{d}\bar{x}}, \tilde{\theta}, \tilde{\theta}_{\mathrm{p}}\right]^{\mathrm{T}}, \tag{21}$$

$$e^{\mathbf{L}\bar{x}} = \begin{bmatrix} e^{\lambda_1\bar{x}} & 0 & 0 & 0 \\ 0 & e^{\lambda_2\bar{x}} & 0 & 0 \\ 0 & 0 & e^{\lambda_3\bar{x}} & 0 \\ 0 & 0 & 0 & e^{\lambda_4\bar{x}} \end{bmatrix}, \tag{22}$$

where $\lambda_i$ $(i = 1, 2, 3, 4)$ are the eigenvalues of the $4 \times 4$ coefficient matrix $\mathbf{A}$,

$$\mathbf{A} = \begin{bmatrix} Pe & 0 & PeA_1 & -PeA_3 \\ 0 & 0 & -A_3/K_{\mathrm{p}} & A_2/K_{\mathrm{p}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{23}$$

The matrix $\mathbf{U} = \{u_{ij}\}$ is a $4 \times 4$ matrix whose columns are the eigenvectors of the corresponding eigenvalues. The coefficient matrix $\mathbf{D}$ is determined by the boundary conditions (15) and (16) and can be expressed as

$$\mathbf{D} = \begin{bmatrix} u_{31} - u_{11}/Pe & u_{32} - u_{12}/Pe & u_{33} - u_{13}/Pe & u_{34} - u_{14}/Pe \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{11}e^{\lambda_1} & u_{12}e^{\lambda_2} & u_{13}e^{\lambda_3} & u_{14}e^{\lambda_4} \\ u_{21}e^{\lambda_1} & u_{22}e^{\lambda_2} & u_{23}e^{\lambda_3} & u_{24}e^{\lambda_4} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\theta}_{\mathrm{in}} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{24}$$

This solution is valid only if $1/Pe > 0$ and $K_{\mathrm{p}} > 0$. With the same procedure it is easy to obtain the solutions for $1/Pe = 0$ or $K_{\mathrm{p}} = 0$ as follows.

(1) $1/Pe = 0$ *and* $K_{\mathrm{p}} > 0$. If the fluid flow in the exchanger is assumed uniform and therefore there is no axial heat dispersion in the fluid, the ordinary differential equation system is of the third-order. The corresponding coefficient matrix of the eigenfunction reduces to

$$\mathbf{A} = \begin{bmatrix} 0 & -A_3/K_{\mathrm{p}} & A_2/K_{\mathrm{p}} \\ 0 & -A_1 & A_3 \\ 1 & 0 & 0 \end{bmatrix}. \tag{25}$$

The coefficient matrix of the boundary condition reduces to

$$\mathbf{D} = \begin{bmatrix} u_{21} & u_{22} & u_{23} \\ u_{11} & u_{12} & u_{13} \\ u_{11}e^{\lambda_{11}} & u_{12}e^{\lambda_{12}} & u_{13}e^{\lambda_{13}} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\theta}_{\mathrm{in}} \\ 0 \\ 0 \end{bmatrix} \tag{26}$$

with

$$\widetilde{\mathbf{Y}} = \left[\frac{\mathrm{d}\tilde{\theta}_{\mathrm{p}}}{\mathrm{d}\bar{x}}, \tilde{\theta}, \tilde{\theta}_{\mathrm{p}}\right]^{\mathrm{T}}. \tag{27}$$

(2) $1/Pe > 0$ *and* $K_{\mathrm{p}} = 0$. If the axial heat conduction in the separating plates is negligible, the resulting ordinary differential equation is of the second-order. The solution can be expressed explicit as

$$\tilde{\theta} = Pe\frac{\lambda_2 e^{-\lambda_1(1-\bar{x})} - \lambda_1 e^{-\lambda_2(1-\bar{x})}}{\lambda_2^2 e^{-\lambda_1} - \lambda_1^2 e^{-\lambda_2}}\tilde{\theta}_{\mathrm{in}}, \tag{28}$$

where

$$\lambda_{1,2} = \frac{1}{2}Pe\left[1 \pm \sqrt{1 + \frac{4}{Pe}(A_1 - A_3^2/A_2)}\right]. \tag{29}$$

The solution for $1/Pe = 0$ and $K_{\mathrm{p}} = 0$ at the outlet of the exchanger $\bar{x} = 1$ has been given by Luo and Roetzel [22]. It can also be obtained from Eq. (28) with $Pe \to \infty$ which yields

$$\tilde{\theta} = \tilde{\theta}_{\mathrm{in}} \exp\left[-\left(A_1 - A_3^2/A_2\right)\bar{x}\right]. \tag{30}$$

Thus the analytical solution of the outlet fluid temperature response in the Laplace plane has been completed.

Special attention should be paid to the calculation if the values of $1/Pe$ and $K_{\mathrm{p}}$ are small, which would yield large values of eigenvalues and cause an irregular operation in exponential function. In the present calculation a complex variable $z$ is expressed with double precision variables $a$, $b$ and $c$,

$$z = (a + \mathrm{i}b)\mathrm{e}^c = \{a, b, c\} \tag{31}$$

where $a = \mathrm{Re}(z)/|z|$, $b = \mathrm{Im}(z)/|z|$ and $c = \ln|z|$.

## 4. Numerical inverse algorithms

Obviously, the analytical inverse transformation of the solution obtained above is difficult or even impossible. Therefore the numerical inverse techniques have to be used to obtain the response in the real-time domain.

Roetzel [24] suggested two algorithms for the inverse transform: Gaver–Stehfest algorithm [25,26] and FFT algorithm [27,28]. Both of them have successfully been applied to the transient analysis in heat exchangers. The Gaver–Stehfest algorithm requires very little computer time. However, it is valid only if there are no oscillatory components and no discontinuities in the region $\bar{\tau} > 0$. Comparing with the Gaver–Stehfest algorithm, the FFT algorithm has no such restrictions and can provide the whole outlet temperature history simultaneously. Therefore it is more suitable for the whole curve matching. However, if there are discontinuity points (e.g., for the case of an inlet step change together with $1/Pe = K_p = 0$ and $B > 0$), the FFT algorithm will produce additional oscillations near the discontinuity point $\bar{\tau} = B$. The maximum magnitude of these additional components could reach about 9% of the magnitude of the real step change [29]. This phenomenon is called Gibss' phenomenon and cannot be eliminated by increasing the number of calculating points. For such a case, Luo and Niemeyer [30] suggested to introduce a very large dispersive Peclet number (e.g., $Pe = 10^6$) into the governing equation system to eliminate the discontinuity point.

## 5. Results and discussions

To discuss the effect of the lateral heat conduction resistance of fins on the outlet fluid temperature response, it is convenience to use the Biot number of fins to represent the results. The Biot number of fins is defined as $Bi = h\alpha_f F_f/(A_f k_f) = \xi \mathrm{NTU}/K_f$, which is the ratio of the lateral heat conduction resistance of fins to the convective heat transfer resistance at the fin surface. $Bi = 0$ means that the temperature in fins is laterally uniformly distributed and is the same as that of the separating plates, therefore it represents a uniformly distributed porous matrix. If $Bi \to \infty$ the lateral temperature distribution in fins is uniform but differs from that of the separating plates and no heat exchange between the fins and the separating plates will occur. It represents a binary porous matrix in which the two solid components have no thermal contact. In a plate–fin heat

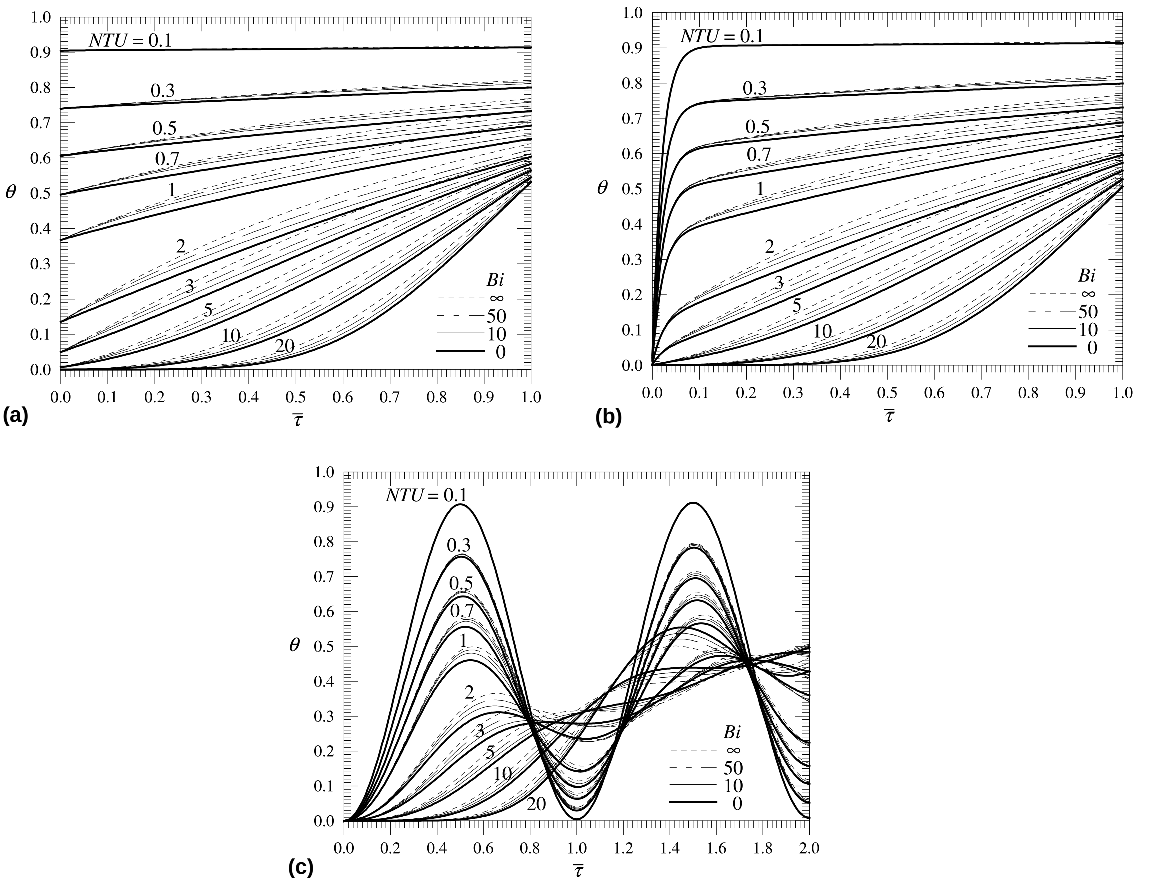


Fig. 2. The effect of the lateral heat conduction resistance of fins on the outlet temperature response, $B = 0, Pe = \infty, K_p = 0, \xi = 0.5, \zeta = 0.2$. (a) $\theta_{in}(\bar{\tau}) = 1$, (b) $\theta_{in}(\bar{\tau}) = 1 - \exp(-\bar{\tau}/0.02)$, (c) $\theta_{in}(\bar{\tau}) = 0.5(1 - \cos 2\pi\bar{\tau})$.

exchanger, because of the lateral heat conduction in fins, the temperature distribution lies between the above two extreme cases. Since the heat transfer between the fluid and the solid matrix is reduced by the lateral heat conduction resistance of fins, as a result, the outlet fluid temperature rises more rapidly than that of the uniformly distributed porous matrix. Such an effect is shown in Fig. 2 for the step change, exponential change and sinusoidal change in inlet fluid temperature, respectively. If the lateral heat conduction resistance of fins is not taken into account, the curve matching would yield a lower value of NTU.

The effect of axial dispersion in the fluid and heat conduction in the separating plates are shown in Figs. 3 and 4, respectively. Both of them will result in an early temperature rise at the outlet of the heat exchanger. Therefore the negligence of the axial dispersion/conduction would also yield a lower value of NTU. Fig. 4 shows further that the effect of the axial heat conduction in the separating plates is significant only for large values of NTU.

The effect of the axial heat conduction in fins should be similar to that in the separating plates. In the present work, the axial heat conduction in fins is neglected. According to the results shown in Fig. 4, this assumption is valid for small values of NTU. If the fin surface is not continuous in the flow direction, such as offset fins and lamellar fins, the effect of the axial heat conduction in fins are also negligible.

To illustrate the influence of the lateral heat conduction resistance of fins, the experiments made by Luo and Cai [31], in which three plate–fin heat exchangers made up of stainless steel were measured using the single-blow testing technique without considering the lateral heat conduction resistance of fins, are reanalysed with the model developed in this work. The parameters of the heat transfer surfaces they used are listed in Table 1. The temperature deviations between the plate–fin heat exchanger model and the uniform porous matrix model are calculated at the optimal matching time at which the uncertainty of NTU has the lowest value. In Table 1 the relative errors of evaluated values of NTU

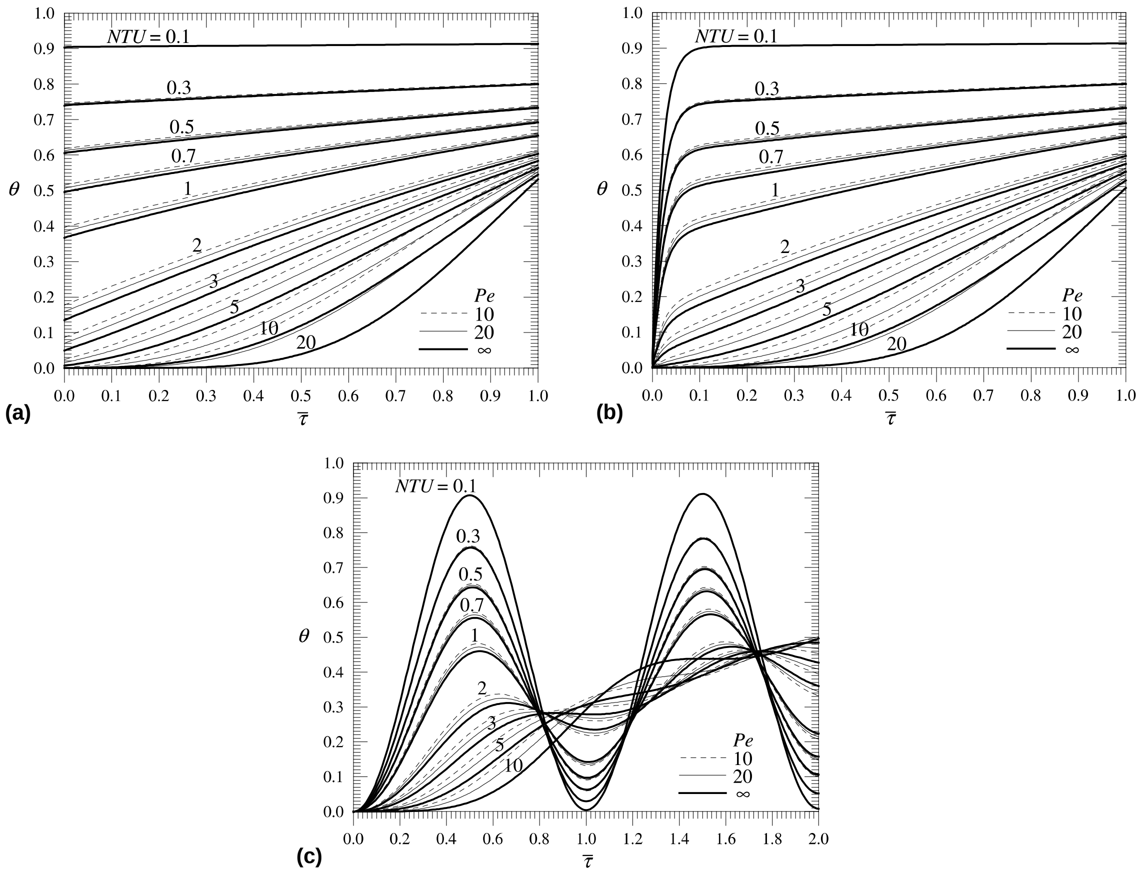


Fig. 3. The effect of the axial dispersion in the fluid on the outlet temperature response, $B = 0, K_p = 0, Bi = 0, \xi = 0.5, \zeta = 0.2$. (a) $\theta_{in}(\bar{\tau}) = 1$, (b) $\theta_{in}(\bar{\tau}) = 1 - \exp(-\bar{\tau}/0.02)$, (c) $\theta_{in}(\bar{\tau}) = 0.5(1 - \cos 2\pi\bar{\tau})$.

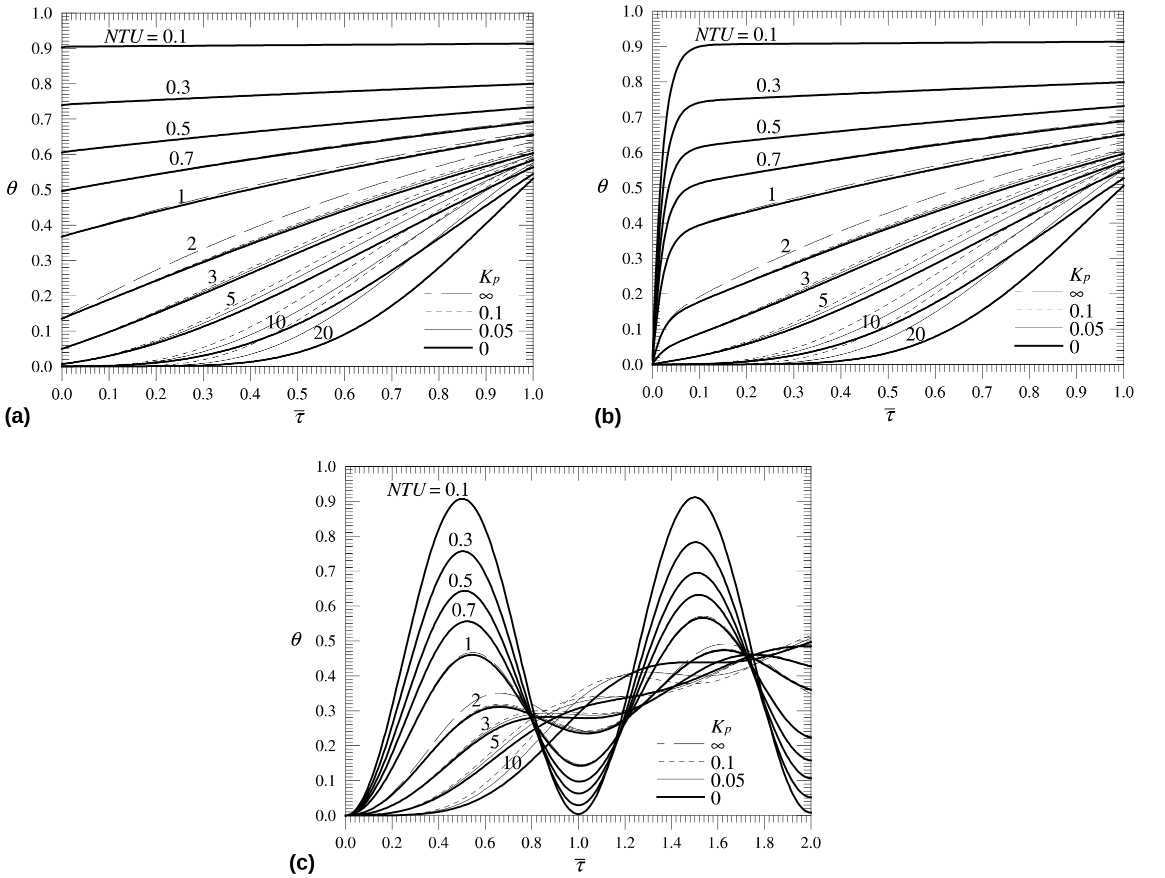


Fig. 4. The effect of the axial heat conduction in the separating plates on the outlet temperature response, $B = 0$, $Pe = \infty, Bi = 0, \xi = 0.5, \zeta = 0.2$. (a) $\theta_{in}(\bar{\tau}) = 1$, (b) $\theta_{in}(\bar{\tau}) = 1 - \exp(-\bar{\tau}/0.02)$, (c) $\theta_{in}(\bar{\tau}) = 0.5(1 - \cos 2\pi\bar{\tau})$.

Table 1
Parameters of three stainless steel plate–fin heat exchangers tested by Luo and Cai [31] and their relative errors of NTU if the lateral heat conduction resistance of fins is neglected[a]

| No. | $h$ (mm) | $s$ (mm) | $\xi$ | $\zeta$ | NTU | $Bi$ | $\Delta\theta$ | $\Delta\text{NTU}/\text{NTU} \times 100\%$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 3.08 | 2.89 | 0.5168 | 0.1896 | $9.07 \sim 3.84$ | $0.151 \sim 0.467$ | $0.0004 \sim 0.0015$ | $-0.26 \sim -0.78$ |
| 2 | 9.58 | 3.27 | 0.7514 | 0.3996 | $4.60 \sim 2.11$ | $1.056 \sim 5.035$ | $0.0025 \sim 0.0093$ | $-1.39 \sim -3.68$ |
| 3 | 9.39 | 2.26 | 0.8141 | 0.4855 | $8.88 \sim 3.10$ | $1.390 \sim 5.750$ | $0.0023 \sim 0.0105$ | $-1.46 \sim -5.21$ |

[a] $L = 600$ mm, $\delta_p = 0.5$ mm, $\delta_f = 0.15$ mm, $k = 29.4$ W/m K, $\rho = 7830$ kg/m$^3$.

neglecting the lateral heat conduction resistance are estimated under the step change inlet condition, which also results the lowest values of NTU uncertainty as has been discussed by Roetzel and Luo [32]. Even so the relative error of NTU of test core No. 3 reaches to −5.21%. The actual values of the relative error might be larger than those given in Table 1. The present analysis explains why the evaluated values of NTU of test core No. 1 given by Luo and Cai [31] were close to the numerical results under constant heat flux condition but those of test cores Nos. 2 and 3 were close to the numerical results under constant wall temperature conduction which is lower than the former. This example tells us that the assumption of uniform porous medium is no longer valid for the plate–fin heat exchanger with lower fin efficiency.

## 6. Conclusions

The traditional single-blow technique is based on the assumption that the heat transfer surface to be tested can be considered as a uniformly distributed porous matrix. This assumption might not be valid for plate-fin heat exchangers. In plate–fin heat exchangers the fins and the separating plates have different temperature dynamic behaviours, which yields lateral heat conduction in fins. In the present work, the effects of the lateral heat conduction resistance along the fins, the axial heat conduction in the separating plates and the axial thermal dispersion in the fluid are investigated. A new mathematical model for the single-blow transient testing technique is developed to take these effects into account by solving the temperature dynamics in the fluid, separating plates and fins simultaneously.

Since the plate–fin heat exchangers made up of aluminium have low values of *Bi* (high fin efficiency), the effect of the lateral heat conduction resistance is usually negligible. However, the plate–fin heat exchangers made up of stainless steel usually have higher values of *Bi* (lower fin efficiency). For such heat exchangers the lateral heat conduction resistance of fins cannot be neglected, otherwise the evaluated values of NTU would be lower than their real values.

In the present work, the axial heat conduction in fins is neglected. In some cases, such as the fins with continuous surface and high values of height-to-space ratios, this assumption might not be valid. For such cases a numerical procedure could be used to account for the axial heat conduction in fins.

The mathematical model developed here is a general model and can be applied to usual heat exchangers. By setting $\zeta = \xi = 0$ (no fins in the exchanger), the present model reduces to the conduction/dispersion model [17].

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